

# Downsian competition when no policy is unbeatable

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## Abstract

This paper studies two-party electoral competition in a setting where no policy is unbeatable. It is shown that if parties take turns in choosing policy platforms and observe each other's choices, for one party to change position so as to win is pointless since the other party never accepts an outcome where it is sure to lose. If there is any cost to changing platform, the prediction is that the game ends in the first period with the parties converging on whatever platform the incumbent chooses. If, however, there is a slight chance of a small mistake, the incumbent does best in choosing a local equilibrium platform. This suggests that local equilibrium policies can be the predicted outcome even if the voting process is not myopic in any way.

*Keywords: Political theory; Voting; Downsian competition; Local equilibrium; Spatial trembles.*

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# 1 Introduction

When Anthony Downs in his classic book *An economic theory of democracy* (1957) considered electoral competition he concluded that, in settings where there exists no “Condorcet winner” and, hence, no policy is unbeatable, incumbent parties always lose elections. The reason for this is simple: no matter what policy an incumbent advocates, there is always some other policy which is majority preferred to it. If this policy is chosen by a challenger, he can be sure to win the election. In Downs’ words:

*The opposition need only [...] wait for the government to commit itself [...]. Then it merely selects the policy that defeats whatever the government has chosen, and - presto! - it is elected!*<sup>1</sup>

But what if the incumbent government is allowed to react to the policy announcement of the opposition? Why would they then stick to their losing policy? After all, whatever the challenger has chosen, there is, by definition, always some other policy which defeats that choice. If the incumbent government were to move to this point, it would again be in a winning position. There may, of course, be costs associated with such a change, but it must surely be better to move than to passively wait to be defeated.

This paper studies a pre-election game with precisely these features. Two parties, an incumbent and a challenger, choose policy platforms sequentially. They observe each others’ announcements and whenever one of them has made a choice, the other can respond either by altering his position or by keeping to his initial choice. If either party chooses to stop, the policy platforms last chosen will be the positions held by the parties in the election.

This modified version of Downsian electoral competition is very similar to the Rubinstein-Ståhl bargaining game, with the important difference that the possible outcomes are indivisible. Parties can either be positioned so as to represent a winning policy, a losing policy, or one where both parties have equal probability of winning the election. This leads to the game having many subgame perfect equilibrium

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<sup>1</sup>Downs (1957), p.62. The underlying problem of voting in a setting without an unbeatable policy was first studied by Condorcet (1785) and later “rediscovered” and formalized by Black (1948) and Arrow (1951) leading Downs to call this situation an “Arrow-problem”.

outcomes. However, after iterated elimination of weakly dominated actions, the game ends after the first round with both parties advocating the platform initially chosen by the incumbent and with both parties having an equal chance of winning the election.<sup>2</sup>

In terms of predicting an equilibrium policy, this suggests inertia. Any policy which has been picked by the incumbent government, for whatever reason, is likely to remain in place after the election. As such, this paper gives an answer to Gordon Tullock's (1981) question about why there seems to be such a contrast between the theoretically predicted policy cycling and the observed stability in actual politics.<sup>3</sup> The simple reason suggested here is that "entering a cycle" is costly but never increases the chances of winning. In the absence of a Condorcet winner no matter which policy a party chooses, his opponent always has the same responses available: play so as to win, so as to lose, or so as to having an equal chance of winning the election. If no party accepts losing, and continued play is costly, immediate convergence is preferred.

However, the above reasoning does not single out any particular policy as being more likely than the next. The incumbent can choose *any* initial policy; given that players only care about winning, the opponent does best in choosing the same policy at once. This naturally raises the question: is there a best initial choice for the incumbent and, if so, what characterizes such a policy?

What is argued in this paper is that when taking the possibility of mistakes into account, a *local equilibrium* policy turns out to be the best initial choice for the incumbent. This choice is best since it maximizes the likelihood of winning the election if the opponent makes a mistake. This result relies on the following interpretation of what it means to make a mistake in this kind of spatial game: If

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<sup>2</sup>This seems a reasonable refinement given the relations between the elimination of dominated strategies and the uncovered set (sequential elimination of dominated strategies leading to the iterated uncovered set). See, for example, Dutta and Laslier, 1999.

<sup>3</sup>The large literature concerned with the question of stability under the Condorcet paradox. One is to consider mixed strategies (see Laffond, Laslier and LeBreton (1993), and Laslier (2000) for an interpretation of this) or uncertainty of some kind, as in probabilistic voting models. Another, following Riker (1980) is to induce more structure and an explicit institutional framework. See also Persson and Tabellini (2000) pp 32-40 for an overview. The rationale for stability suggested here is quite different from these.

a party is to choose a platform represented by a point on the real line, a *tremble* will lead the party to choose a point close (in terms of distance) to the one aimed for. To be very explicit, if a party attempts to pick a particular policy (often a tax rate in applications) between 0 and 1 as their platform, say 0.33, a tremble may lead them to accidentally choose 0.32 or 0.34 (or generally  $0.33 \pm \varepsilon$ ) instead. The idea is clearly related to refinements such as Selten's (1975) perfect equilibrium and Myerson's (1978) proper equilibrium, but it is not the same. The *spatial tremble* suggested here implies that the likelihood of the mistake is related to the distance from the point aimed for, and not to the consequences when making the mistake, even though these are of course taken into account when deciding on a strategy.<sup>4</sup> The intuition for this spatial interpretation of a tremble is that if the points on the line corresponds to actual policies, like a tax rate in many applications, it is somewhat difficult to see how an actual mistake, when trying to pick 0.33, could result in accidentally picking, say 0.58.<sup>5</sup> In contrast, mistakes leading to  $0.33 \pm \varepsilon$  are relatively easy to envision.

That local equilibria emerge as the predicted outcome is interesting for at least three reasons. Firstly, a local equilibrium exists under very general conditions when the alternative set is one dimensional, as shown by Kramer and Klevorick (1974).<sup>6</sup> Secondly, there are numerous examples of one dimensional political-economy problems where the existence of a global equilibrium can not be guaranteed in general.<sup>7</sup> These have typically been dealt with by restricting the problem to cases where a global equilibrium exists, (an exception is a recent paper by Crémer and Palfrey (2002)). Thirdly, as was first shown by Grandmont (1978), even if the policy space

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<sup>4</sup>If, for example, a first best strategy prescribes choosing 0.33, while the second best choice is 0.55 and the third best 0.29 in a game without mistakes, a proper equilibrium refinement would place the most weight on the second best choice and less on the third best. In contrast 0.29 is more likely with a spatial tremble since it is closer. Noting this difference may also be of some general interest.

<sup>5</sup>It is important to note that what is considered here is the possibility of a mistake, not uncertainty. The model developed is one of perfect information.

<sup>6</sup>As is well known, if the alternative set is multidimensional local equilibria exist only under very extreme assumptions, as shown by Plott (1967). See Austen-Smith and Banks (1999) on a thorough discussion of the subsequent literature on this.

<sup>7</sup>A classical example of such problems are those dealing with the public provision of private goods, as noted by Musgrave (1956), and Stiglitz (1974), and more recently studied in e.g. Epplé and Romano (1996a), (1996b), and Fernandez and Rogerson (1995).

is multidimensional, voter conflict can still be viewed as one dimensional when preferences can be projected on a unidimensional space in which different voters can be ordered by their type.<sup>8</sup> In all such cases local equilibria also become relevant.

The main reason for previous reluctance toward the local equilibrium concept has been the belief that for it to be interesting it “requires the voting process to be myopic”. To quote Atkinson and Stiglitz (1980): “whether it provides a persuasive resolution to the “majority-voting paradox” depends on the extent to which choices are limited to small perturbations of the existing situation”.<sup>9</sup> More recently renewed interest has been shown for the local equilibrium concept in, for example, Alós-Ferrer and Ania (2001), and Crémer and Palfrey (2002). This paper shows that a local equilibrium is the predicted outcome in a reasonable extensive form game without any such myopic restrictions.

Wuffle et al. (1989) study a situation very similar to the one considered in this paper, namely the outcome of a two-candidate sequential competition in an  $n$ -dimensional majority rule spatial voting game without a core. Their claim is that in the absence of a Condorcet winning policy, an incumbent will locate at the “Finagle point”. This position has the property that every point in the space is defeated by some point very close to the Finagle point. This implies that no matter what a challenger suggests, the incumbent does not have to change his policy by much to counter the challenge. The main reason for advocating such a point lies in the assumption that altering a policy is costly, and that this cost increases with the spatial distance moved. Wuffle et al. do not, however, explicitly model the game, leading to that some problems, such as motivating who will be the “last mover”, remains.<sup>10</sup> In contrast, this paper explicitly studies a formally specified pre-election game. The fact that changing platforms is costly is modeled in the simplest possible way, namely that both candidates have a cost for each round of

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<sup>8</sup>What in Grandmont (1978) is called “intermediate preferences”.

<sup>9</sup>Atkinson and Stiglitz (1980), p. 307.

<sup>10</sup>They have examples where they argue that locating at the Finagle point may be advantageous both when the incumbent moves last, as well as when he does not. In the latter case, the argument relies on the attractiveness of being positioned at the Finagle point in subsequent elections, even though losing the “current one”.

continued campaigning. As will be shown, this results in different predictions.

Most of the literature which has modified and extended Rubinstein's bargaining game to a political setting has used it to study legislative bargaining situations (such as Baron and Ferejohn (1989), Baron (1994) and Baron (1996)) with explicit proposal, amendment and voting stages. Other studies that have taken a sequential choice theoretic approach to electoral competition (following e.g. Kramer (1977)) have considered sequential elections. What is suggested here is much closer to Rubinstein's original model and differs from legislative bargaining games in that it studies electoral competition between two candidates. In contrast to models of sequential elections, the sequential game considered here takes place before an election.

## 2 The Game

Consider a situation where two parties,  $A$  and  $B$ , are to choose their policy platforms,  $x^A$  and  $x^B$ , before an election. The set of alternatives is such that  $x^i \in [0, 1]$ . The underlying preferences of the electorate are such that every policy,  $x$ , has a relation to every other policy,  $x'$ , such that  $x \succ x'$ , ( $x$  is majority preferred to  $x'$ ), or  $x' \succ x$ , ( $x'$  is majority preferred to  $x$ ), or  $x \sim x'$  (an equal share of the population supports the respective policies or equivalently, everyone is indifferent between  $x$  and  $x'$ ). For simplicity, I assume that only the same policy has this last property, i.e.  $x \sim x'$  only if  $x = x'$ . Furthermore, the situation is such that there is no unbeatable policy, that is, for all  $x$  there exist  $x' \succ x$ , (i.e. there is no Condorcet winner). There is, however, at least one local equilibrium policy,  $x^*$ , such that it is majority preferred to any neighboring policy, with simplified notation,  $x^* \succ x^* \pm \varepsilon$ .<sup>11</sup> The consequences for the election outcome is obviously that if  $x^A \succ x^B$  at the time of the election, then  $A$  wins with certainty, if  $x^A \prec x^B$ ,  $B$  wins with certainty, and if  $x^A = x^B$ , both face an equal chance of winning. Parties are assumed to only care about winning the election, (as implied by the fact that they are Downsian candidates), not about

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<sup>11</sup>More precisely,  $x^*$  is a local equilibrium policy if there exist  $\eta > 0$  such that for all  $x$  such that  $|x - x^*| < \eta$ ,  $x^*$  is majority preferred to  $x$ .

the margin by which they win or about the policy implemented.

Before the election, candidates can alter their platforms any number of times.<sup>12</sup> Party  $A$ , the incumbent, makes a first choice,  $x_0^A$ . Party  $B$ , the challenger can now choose a competing platform,  $x_0^B$ , which is such that either  $x_0^B \succ x_0^A$ , or  $x_0^B \prec x_0^A$ , or  $x_0^B = x_0^A$ . After observing this choice, party  $A$  can choose to alter his position or “stop the game”. If  $A$  stops, the election is held with parties representing platforms  $x_0^A$  and  $x_0^B$  respectively. If  $A$  moves, he picks a new platform  $x_1^A$  which is observed by  $B$ , who then has the option of stopping or continuing one more period, etcetera. Continuing the game has a cost for both players, (which can be thought of as a cost of continued campaigning), captured by the outcomes being discounted by  $\delta \in (0, 1)$  in each period,  $t \in \{0, 1, 2, \dots\}$ .

The situation is obviously analogous to a bargaining game with alternating offers, as in Rubinstein (1981). Figure I shows the structure of the game and the payoffs from different outcomes.

However, unlike the traditional alternating offer game, the continuum of platform choices does not correspond to a continuum of outcomes. After party  $A$  has made the initial choice, only three responses in terms of relevant strategic choices are available for party  $B$  : either  $B$  chooses a platform which is majority preferred to  $A$ 's initial policy, or a policy such that  $A$ 's initial policy is majority preferred to his choice, or one where the population is split between the two parties. The situation is the same whenever a player who is called upon to move chooses to continue the game. This strategic simplification of the game is possible since the candidates only care about winning or losing, not by how much, and not about which policy is implemented.

## 2.1 The equilibrium outcome without mistakes

Unlike Rubinstein's game, which has a unique subgame perfect equilibrium, this game has many subgame perfect equilibria, due to the indivisibility of outcomes.

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<sup>12</sup>One may think of this as capturing the fact that “there is always some time left before the election” and hence, no party can expect to have the last move. See e.g. Myerson (1991) for a discussion on the interpretation of an infinite horizon in bargaining games.

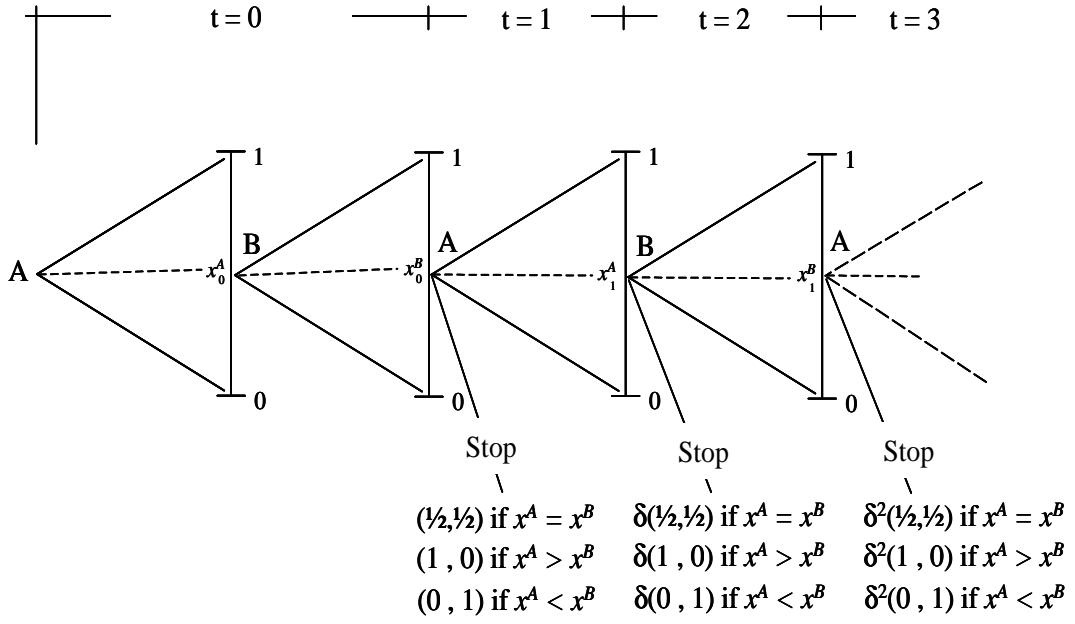


Figure 1: The pre-election game with alternating platform choices and no unbeatable policy.

There is, however, only one type of play that survives an iterated elimination of weakly dominated actions. This gives subgame perfect equilibria which predict that the game ends in the first period with both parties converging on the platform initially chosen by party A, with both parties having an expected gain of  $\frac{1}{2}$ .

**Proposition 1** *In the only subgame perfect equilibrium that survive the elimination of weakly dominated strategies, a player  $i$ , who has the choice of stopping or continuing the game, stops whenever  $x^i \succeq x^j$  and continues otherwise.*

**Proof.** First, note that the game is stationary after the initial choices, that is, the subgame starting in period 1, (3, 5, etc.) is identical to that starting in 2 (4, 6, etc.), with the players' roles reversed. This means that party A faces the same choices in every odd numbered period, while party B faces the same situation in every even numbered period. Given this, the following is true for any subgame:

i) If given the option to stop when facing a certain win, this is always optimal since no continuation of the game can give a higher payoff (in any period  $t$ , the



value of winning with certainty is  $\delta^t$ , while the highest possible continuation value is  $\delta^{t+1} < \delta^t$ ).

ii) If given the option to stop when facing a certain loss, this is at least weakly dominated, since no continuation of the game can give a lower payoff than zero. Hence, no strategy profile involving stopping at a certain loss survives an iterated elimination of weakly dominated strategies.

iii) Finally, faced with a fifty percent chance of winning, stopping has an expected value of  $\delta^t/2$  in any period  $t$ . All profiles involving continuing and accepting a loss in the future are obviously worse since  $0 < \delta^t/2$ . The outcome  $\frac{1}{2}$  in a future period  $t+s$  is also worse, since  $\delta^{t+s}/2 < \delta^t/2$ . So only a profile resulting in a certain future win can be better than stopping at once (this would be the case whenever  $\delta^{t+s} > \delta^t/2$ ). However, no such profiles exist by ii), since the opponent never stops in  $t+1$  when facing a certain loss. Hence, it is always optimal to stop when facing a fifty percent chance of winning.

This implies that a player  $i$ , at any point  $t$ , chooses to stop when  $x^i \succeq x^j$ , while  $i$  always continues if  $x^i \prec x^j$ . Consequently, a player  $j$  who “responds” to the platform chosen by  $i$ , takes the position  $x^j = x^i$ , which is “accepted” by player  $i$ . ■

Even if there is no majority-rule equilibrium, in a social-choice theoretic sense, this modified version of Downsian electoral competition, with alternating platform choices, predicts the following equilibrium outcome:

**Proposition 2** *The equilibrium outcome of the pre-election game, where candidates take turns in choosing their policy platform, is that both candidates immediately converge on the policy suggested by the incumbent. Both candidates have an equal chance of winning the election.*

**Proof.** Follows immediately from Proposition 1. ■

In terms of predicting an outcome, this suggests that policies are likely to remain unchanged. Whatever policy is initially picked by the incumbent will also be the outcome of the game. Player  $A$  can choose *any* policy,  $x_0^A$ , in the initial round and

$B$  will choose  $x_0^B = x_0^A$ , after which the game ends.<sup>13</sup>

## 2.2 The equilibrium outcome with spatial trembles

Even though the above, for a very simple reason, predicts policy stability and that parties have equal chances of winning elections, as oppose to cycling and incumbents always loosing, the prediction that any policy can be the outcome is clearly unappealing. Surprisingly, introducing a small probability that parties make mistakes in their platform choices narrows down the number of equilibrium outcomes to parties converging on a *local-equilibrium* policy.

What does it mean to “make a mistake” when choosing a platform? The suggestion here is that when deciding on a location in a policy space, a mistake should have a spatial interpretation. In a setting where similar policies are also close in distance, it seems natural that a small mistake should lead to accidentally choosing a platform close to the one aimed for. In terms of the analogy often used when explaining “trembling-hand perfect equilibria”, a mistake can literally be described as the result of a hand trembling slightly when trying to point out a position on a line (or in space). The suggested equilibrium is, hence, a “spatial trembling-hand perfect equilibrium”. With this interpretation of trembles, it turns out that a local equilibrium policy has the property of maximizing the likelihood of winning in case the opponent trembles, leading to it being a best initial choice for the incumbent.

To simplify the analysis we assume that the incumbent does not make a mistake in his initial choice, which is also reasonable in this setting. Being the incumbent means having ample time to position oneself before any challenger appears. It is also assumed that mistakes only take the form of accidentally choosing neighboring points to the one aimed for.<sup>14</sup>

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<sup>13</sup>One can also note that there is nothing in the proof which is specific to a one-dimensional setting. The prediction of immediate convergence can easily be extended to an  $n$ - dimensional policy space.

<sup>14</sup>This is also a simplifying assumption. In general what is important is that it is more likely that a mistake results in the choice of a point close in distance, rather than in the choice of a more distant point. This, as well as the allowing for the possibility that the incumbent trembles, will be discussed in the next section.

In terms of the above game  $A$ , being the incumbent, chooses the initial platform,  $x_0^A$  with certainty. However, when the challenger,  $B$  is to choose  $x_0^B$ , he may by mistake play  $x_0^B \pm \varepsilon$ , which happens with some positive probability  $p$ , ( $x_0^B + \varepsilon$  with probability  $p/2$  and  $x_0^B - \varepsilon$  with probability  $p/2$ ). If  $A$  chooses to continue after having observed  $B$ 's choice he no longer has the advantage of having had time to position himself and may also tremble around his choice  $x_1^A$ , and may by mistake play  $x_1^A \pm \varepsilon$ , with probability  $p$ . Crucially, it is assumed that the probability of mistakes is so small that it does not alter the optimal play given by Proposition 1, that is when player  $i$  is called upon to move he stops whenever  $x^i \succeq x^j$  and continues otherwise. The assumption needed for this to be the case is that the expected value of continuation is smaller than the expected value of immediate convergence, even when one may make a mistake, i.e.  $EV(\text{continue}) < \delta^t \frac{1}{2}(1-p) \forall t$ . This is certainly true for some positive  $p$  as  $p \rightarrow 0$ .<sup>15</sup>

**Proposition 3:** *If there is a small positive probability that the challenger  $B$  trembles in his initial choice of platform, then it is optimal for the incumbent  $A$  to initially choose a local-equilibrium policy platform  $x^*$ . The global policy outcome will be this local-equilibrium policy, and party  $A$  has a slightly better chance of winning the election than party  $B$ .*

**Proof.** First, note that the game is still a game of complete information and that the optimal strategy given in Proposition 1 is valid, that is, whenever a player  $i$  is to move or to stop, it is optimal to stop if  $x^i \succeq x^j$ , while  $i$  always continues if  $x^i \prec x^j$ . As in the case above, the best continuation is always  $x_{t+1}^i = x_t^j$ .

Consider first any choice of initial policy,  $x_0^A$ , which is not a local equilibrium, (that is  $x_0^A \neq x^*$ ).  $B$ 's best response is to choose the same policy  $x_0^B = x_0^A$ . If  $B$  makes a mistake and instead chooses  $x_0^B = x_0^A \pm \varepsilon$  with probability  $p$ , this will, with

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<sup>15</sup>Unlike the previous assumptions this is not a simplifying one, but rather crucial. Without this assumption the optimal play would be determined by a combination of the likelihood of mistakes and the consequences of a mistake. Assume for example that a player would like to converge (this has the highest payoff in the absence of mistakes) but faces a situation where, if he makes a mistake, he loses with certainty. He may then prefer to play in a way which leads to a continuation of the game if the probability of a mistake is large enough. This would change the whole premises of the game.

probability  $p/2$ , lead to a situation where  $x_0^B \prec x_0^A$  and  $A$  stops the game facing a certain win, but it can also, with equal probability  $p/2$ , lead to a situation  $x_0^B \succ x_0^A$ , where  $A$  continues the game. The expected value of choosing any policy  $x_0^A \neq x^*$  is  $EV(x_0^A \neq x^*) = (1 - p)\frac{1}{2} + \frac{p}{2} + \frac{p}{2}EV(\text{continue}) = \frac{1}{2} + \frac{p}{2}EV(\text{continue})$ , where  $EV(\text{continue}) < 1$  since the maximum possible continuation value is  $\delta < 1$ .

If  $A$  instead chooses the local equilibrium policy, *any* tremble from  $B$  will yield a situation where  $x_0^B \prec x_0^A$  since, by the definition of a local equilibrium, it is such that  $x^* \succ x^* \pm \varepsilon$ . This implies that the expected value for party  $A$  from choosing the local equilibrium is strictly greater than choosing any other initial platform, since  $EV(x_0^A = x^*) = (1 - p)\frac{1}{2} + \frac{p}{2} + \frac{p}{2} = \frac{1}{2} + \frac{p}{2} > EV(x_0^A \neq x^*)$ .

$B$ 's best response is to choose  $x_0^B = x_0^A = x^*$ , which means that the game will end after the initial choices since  $A$ 's optimal response is to stop if  $x_0^B = x_0^A = x^*$  as well as if  $x_0^B = x_0^A \pm \varepsilon = x^* \pm \varepsilon$ . This means that the game always ends with the local equilibrium policy  $x^*$  being implemented and  $A$  having a slightly higher possibility of winning. ■

By the same logic as for propositions one and two, the prediction is still that the candidates immediately converge to the same policy. However, having the advantage of moving first, the incumbent chooses a locally “unbeatable” point and positions himself so that any mistake from the challenger leads to a situation which is favorable to the incumbent.

### 3 Discussion

The analysis in the previous section broadly suggest two things about two party competition in a setting where no policy platform is unbeatable.

The first is that, even if the possibility of cycling over policies exist in terms of the underlying preferences of the voters, whether “the cycle is played” by the parties depends on the costs and consequences of such play. In particular, if changing platforms is costly, without leading to an increased probability of winning a subse-

quent election, parties do best in not engaging in such costly cycling behavior. The consequence of this is that the opposition does best in trying to mimic whatever policy platform the incumbent party advocates. This would also suggest that an incumbent party can move the policy anywhere if it does so well before the election, the opposition still does best in copying their platform in the pre-election phase.

Second, if the incumbent can position himself at any policy before an election, knowing that the opposition will try to copy his platform, he does best in locating at a point which is a local equilibrium, since any mistakes on behalf of the opposition will lead to the incumbent winning the election.

There are a number of ways in which the analysis could be extended without changing these results.

First, the local equilibrium prediction does not hinge on the incumbent not making mistakes. Even if there is a small possibility that the incumbent makes a mistake when announcing his first choice, he still does best in aiming for a local equilibrium policy. To see this, consider a game-tree with both players trembling to either side of the point they aim for with probability  $\frac{\epsilon}{2}$ , and recall that each player always tries to mimic the platform of the previous one. If the incumbent aims for a point which is a local equilibrium there are five ways in which the game can still end in the first period: Either both parties manage not to make a mistake in which case the expected outcome is  $(\frac{1}{2}, \frac{1}{2})$ , or  $A$  misses the point he aimed for (to the right or to the left) but  $B$  does not, also leading to the outcome  $(\frac{1}{2}, \frac{1}{2})$ , or  $A$  does not miss but  $B$  misses (to the right or to the left), in which case both mistakes lead to  $A$  winning since the initial choice was a local equilibrium. Now the difference if  $A$  was to choose a point which is not a local equilibrium is, analogous to the proof of Proposition 3, that if  $A$  manages not to make a mistake but  $B$  misses, only one of these outcomes leads to  $A$  winning while the other leads to continued play, which is obviously worse since no continuation is preferred to winning at once. (In both cases there is, of course, also the possibility that both parties make mistakes in their implementation. The outcome of such continuations are specific to each setting

and can not be compared in general, and are therefore assumed equal in the above comparison. Even though very unlikely, parties could in principle keep missing what they aim for, with the consequence that policy moves far from the initial point).

A second observation is that even if a player is allowed to revise a choice which accidentally resulted in a mistake, this does not change the predictions about policy outcomes, it only increases the probability of both parties ending up with the same platform.

Third, with the simplifying assumption that trembles are restricted to points close to the one aimed for, there is no difference between different local equilibrium points, all are equally “resistant to trembles”. In particular, in a setting where there is a local, as well as a global equilibrium policy, both would be equally attractive initial choices. This would, however, not be the case if it is possible to tremble over the whole policy space. In general, assuming that the probability of mistakes is such that points close by are more likely than those further away, but all points can be chosen with positive probability. Then a global equilibrium platform - if one exist - is preferable since no continuation can lead to its’ defeat while a local equilibrium does not have this feature.

## 4 Conclusion

This paper has suggested a new way of pinning down the outcome of a classical Downsian two-candidate competition without a Condorcet winner. In contrast to “the continual defeat of incumbents” suggested by Downs (1957), and the “perpetual change of platforms” in the absence of an unbeatable policy, the suggested interpretation has been the following: Knowing that no party can ever find an unbeatable position, the candidates may as well converge to the same platform at once, since continually changing platforms (to be in a winning position) is costly and the probability of winning can never exceed one half anyway. It has been shown that such reasoning is indeed an equilibrium outcome in an “alternating-platform” pre-election game.

While this result predicts direct convergence to the policy chosen by the incumbent, it says nothing about which policy will be chosen initially. The “refinement” suggested here is that a local-equilibrium platform is stable in a specific sense. A local equilibrium platform beats any neighboring policy and is therefore an optimal initial choice for the incumbent party, whenever there is a small probability that the challenger makes a (spatial) mistake when trying to mimic the incumbent’s platform. The prediction is therefore that the outcome will be a local equilibrium policy, even though the candidates are farsighted and can choose their platforms without any restrictions. As such, this paper presents a defense of the local majority rule equilibrium-concept, first suggested by Kramer and Klevorick (1974), and shows that the assertion by Atkinson and Stiglitz (1980) that “whether it provides a persuasive resolution to the “majority-voting paradox” depends on the extent to which choices are limited to small perturbations of the existing situation” need not be true.

The motivation for many of the papers following Downs (1957) has been the sharp contrast between the disequilibrium results in the models, and the relative inertia in actual politics. As Tullock (1981) phrased it, “Why so much stability?”. This paper predicts policy stability in that candidates immediately converge to the same platform. The very absence of a Condorcet winning policy makes any attempt from a candidate to win pointless, since any position can always be beaten. Entering a “cycle” is costly but does not increase the chances of winning. The paper also predicts a slight advantage for the incumbent, since he can choose to position himself so as to win every time the challenger makes a mistake in his implementation. These two predictions are consistent with two real-world observations, namely stability of policy and the electoral advantage of incumbents.

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